BLOOM | BSO Bloom School Olympiad | GRADE 11

Bloom Mathematics Olympiad
Sample Paper

Maximum Time : 60 Minutes | **Maximum Marks : 60**

INSTRUCTIONS

1. There are 50 Multiple Choice Questions in this paper divided into two sections :

   Section A 40 MCQs; 1 Mark each

   Section B 10 MCQs; 2 Marks each

2. Each question has Four Options out of which **ONLY ONE** is correct.
3. All questions are compulsory.
4. There is no negative marking.
5. No electric device capable of storing and displaying visual information such as calculator and mobile is allowed during the course of the exam.

Roll No.

Student's Name

Section-A (1 Mark each)

1. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is .....

(a) 28 (b) 112 (c) 7 (d) 4

2. In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all of three newspapers, then the number of families which buy A only, is

(a) 4400 (b) 3300 (c) 2000 (d) 500

3. Universal set, $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$, $A = \{x : x^2 - 5x + 6 = 0\}$ and $B = \{x : x^2 - 3x + 2 = 0\}$. Then, $(A \cap B)'$ is equal to

(a) $\{1, 3\}$ (b) $\{1, 2, 3\}$ (c) $\{0, 1, 3\}$ (d) $\{0, 1, 2, 3\}$

4. The set of all real x satisfying the inequality $\dfrac{3 - |x|}{4 - |x|} > 0$

(a) $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$ (b) $(-\infty, -4) \cup (4, \infty)$

(c) $(-\infty, -3) \cup (4, \infty)$ (d) $(-3, 3) \cup (4, \infty)$

5. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(a) $-4 < x < 0$ (b) $0 < x < 1$ (c) $-100 < x < 100$ (d) $-\infty < x < \infty$

6. If $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \le 2$, then the maximum value of xy, $\forall\ x \ge 0,\ y \ge 0$ is

(a) 2500 (b) 3000 (c) 1200 (d) 3500

7. A man arranges to pay off a debt of ₹ 3600 by 40 annual instalments which are in AP. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid. The value of the 8th instalment is

(a) ₹ 35 (b) ₹ 50

(c) ₹ 65 (d) None of these

8. The sum of the integers from 1 to 100 which are not divisible by 3 or 5 is

(a) 2489 (b) 4735 (c) 2317 (d) 2632

9. In a GP, first term is 1. If $4T_2 + 5T_3$ is minimum, then its common ratio is

(a) $\frac{2}{5}$ (b) $-\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

10. There is only one way to choose real numbers M and N such that, when the polynomial $5x^4 + 4x^3 + 3x^2 + Mx + N$ is divided by the polynomial $x^2 + 1$, the remainder is 0. If M and N assume these unique values, then $M - N$ is

(a) –6 (b) –2 (c) 6 (d) 2

11. z_1 and z_2 are two complex numbers such that $|z_1| = |z_2|$ and $\arg(z_1) + \arg(z_2) = \pi$, then z_1 is equal to

(a) $2\bar{z}_2$ (b) $\bar{z}_2$ (c) $-\bar{z}_2$ (d) None of these

12. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{334} - 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{365}$ is equal to

(a) $1 - i\sqrt{3}$ (b) $-1 + i\sqrt{3}$ (c) $4\sqrt{3}i$ (d) $-i\sqrt{3}$

13. There are 10 persons named $P_1, P_2, P_3, \ldots, P_{10}$. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

(a) 4210 (b) 4200 (c) 4203 (d) 4205

14. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which hall can be illuminated.

(a) $2^{10} - 2$ (b) $2^{10} - 1$ (c) $2^{10} + 1$ (d) None of these

15. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. Find the number of different ways of doing questions.

(a) 779 (b) 781 (c) 780 (d) 782

16. If $A = \{x : x^2 - 5x + 6 = 0\}, B = \{2, 4\}, C = \{4, 5\}$, then $A \times (B \cap C)$ is

(a) $\{(2, 4), (3, 4)\}$ (b) $\{(4, 2), (4, 3)\}$
(c) $\{(2, 4), (3, 4), (4, 4)\}$ (d) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

17. If the coefficient of second, third and fourth terms in the expansion of $(1+x)^{2n}$ are in AP, then $2n^2 - 9n$ is equal to

(a) -7 (b) 7 (c) 6 (d) -6

18. The digit at the unit place in the number $19^{2005} + 11^{2005} - 9^{2005}$ is

(a) 2 (b) 1 (c) 0 (d) 8

19. The middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$, is

(a) $\frac{n!}{((n/2)!)^2}$ (b) $\frac{(2n)!}{((n/2)!)^2}$
(c) $\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot 2^n$ (d) $\frac{1 \cdot 3 \cdot 5 \cdots (2n+1)}{n!} \cdot 2^n$

20. If $\cos A = m \cos B$ and $\cot \frac{A+B}{2} = \lambda \tan \frac{B-A}{2}$, then λ is

(a) $\frac{m}{m-1}$ (b) $\frac{m+1}{m}$ (c) $\frac{m+1}{m-1}$ (d) None of these

21. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(a) 3 (b) 4 (c) 2 (d) 5

22. In a ΔABC, $\angle C = 60^\circ$, then $\frac{1}{a+c} + \frac{1}{b+c}$ is equal to

(a) $\frac{1}{a+b+c}$ (b) $\frac{2}{a+b+c}$ (c) $\frac{3}{a+b+c}$ (d) None of these

23. If two vertices of a triangle are $(-2,3)$ and $(5,-1)$. Orthocentre lies at the origin and centroid on the line $x + y = 7$, then the third vertex lies at

(a) (7, 4) (b) (8, 14) (c) (12, 21) (d) None of these

24. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point (3, 2) is $\frac{7}{5}$.

(a) $3x - 4y + 6 = 0$ and $4x - 3y + 1 = 0$ (b) $3x + 4y + 6 = 0$ and $4x + 3y + 1 = 0$

(c) $3x - 4 - 6 = 0$ and $4x + 3y + 1 = 0$ (d) None of these

25. Two lines are drawn through (3, 4) each of which makes angle of 45° with line $x - y = 2$, then area of the triangle formed by these lines is

(a) 9 sq units (b) 9/2 sq units (c) 2 sq units (d) 2/9 sq unit

26. If the straight line $y = mx$ lies outside the circle $x^2 + y^2 - 20y + 90 = 0$, then the value of m will satisfy

(a) $m < 3$ (b) $|m| < 3$ (c) $m > 3$ (d) $|m| > 3$

27. An equilateral ΔSAB is inscribed in the parabola $y^2 = 4ax$ having its focus at S. If chord AB lies towards the left of S, then side length of this triangle is

(a) $2a(2-\sqrt{3})$ (b) $4a(2-\sqrt{3})$ (c) $a(2-\sqrt{3})$ (d) $8a(2-\sqrt{3})$

28. In the normal at the end of latusrectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity e, passes through one end of the minor axis, then

(a) $e^2(1+e^2) = 0$ (b) $e^2(1+e^2) = 1$

(c) $e^2(1+e^2) = -1$ (d) $e^2(1+e^2) = 2$

29. If the sum of the squares of the distance of a point from the three coordinate axes be 36, then its distance from the origin is

(a) 6 (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) None of these

30. Three vertices of a parallelogram $ABCD$ are $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$. Find the fourth vertex D.

(a) (– 4, – 7, – 6) (b) (4, 7, 6) (c) (4, 7, – 6) (d) None of these

31. If $\lim_{x \to \infty} \left[\frac{x^3+1}{x^2+1} - (ax+b)\right] = 2$, then

(a) $a = 1$ and $b = 1$ (b) $a = 1$ and $b = -1$ (c) $a = 1$ and $b = -2$ (d) $a = 1$ and $b = 2$

32. $\lim_{x \to \infty} \frac{x^4 \cdot \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3}$ equals

(a) 0 (b) – 1 (c) 2 (d) 1

33. $\lim_{n \to \infty} \left(\frac{n^2 - n + 1}{n^2 - n - 1}\right)^{n\,(n-1)}$ is equal to

(a) e (b) e^2 (c) e^{-1} (d) 1

34. If harmonic mean of first 5 observations is $\frac{5}{2}$ and harmonic mean of another 5 observations is $\frac{9}{2}$, then harmonic mean of all 10 observations is

(a) 7 (b) $\frac{45}{14}$ (c) $\frac{101}{36}$ (d) None of these

35. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to – a. Also, by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively, then the value of $a^2 + b^2$ is equal to

(a) 425 (b) 650 (c) 250 (d) 925

36. A die is rolled three times. The probability of getting a larger number than the previous number each time is

(a) $\frac{15}{216}$ (b) $\frac{5}{54}$ (c) $\frac{13}{216}$ (d) $\frac{1}{18}$

37. Out of 13 applicants for a job, there are 8 men and 5 women. It is desired to select 2 persons for the job. The probability that atleast one of the selected persons will be a woman, is

(a) $\frac{5}{13}$ (b) $\frac{10}{13}$ (c) $\frac{14}{39}$ (d) $\frac{25}{39}$

38. The probability that at least one of the events A and B occurs is $\frac{3}{5}$. If A and B occur simultaneously with probability $\frac{1}{5}$, then $P(\overline{A}) + P(\overline{B})$ is

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{6}{5}$ (d) $\frac{7}{5}$

39. If $f(x) = \cos ax + \sin x$ is periodic, then a must be

(a) irrational (b) rational

(c) positive real number (d) None of these

40. The range of the function $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$ is

(a) [0, 3] (b) $[0, \sqrt{3}]$ (c) $(-\infty, \infty)$ (d) None of these

Section-B (2 Marks each)

41. After striking the floor, a certain ball rebounds (4/5)th of height from which it has fallen. Then, the total distance that it travels before coming to rest, if it is gently dropped from a height of 120 m is

(a) 1260 m (b) 600 m (c) 1080 m (d) None of these

42. If $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$, then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$, is

(a) $x^2 - x + 2 = 0$ (b) $x^2 + x - 2 = 0$ (c) $x^2 - x - 2 = 0$ (d) $x^2 + x + 2 = 0$

43. There are m men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of m is

(a) 12 (b) 11 (c) 9 (d) 7

44. The value of x, for which the 6th term in the expansion of $\left\{2^{\log_2 \sqrt{(9^{x-1}+7)}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right\}^7$ is 84, is equal to

(a) 4 (b) 3 (c) 2 (d) 5

45. A spherical balloon of radius r subtends an $\angle\alpha$ at the eye of an observer. If the angle of elevation of the centre of the balloon be β, then height of the centre of the balloon is

(a) $r \operatorname{cosec}\left(\frac{\alpha}{2}\right)\sin\beta$ (b) $r \operatorname{cosec}\alpha \sin\left(\frac{\beta}{2}\right)$ (c) $r\sin\left(\frac{\alpha}{2}\right)\operatorname{cosec}\beta$ (d) $r\sin\alpha \operatorname{cosec}\left(\frac{\beta}{2}\right)$

46. The equations of perpendicular bisectors of sides AB and AC of a ΔABC are $x - y + 5 = 0$ and $x + 2y = 0$, respectively. If the coordinates of vertex A are $(1, -2)$, then equation of BC is

(a) $23x + 14y - 40 = 0$ (b) $14x - 23y + 40 = 0$ (c) $23x - 14y + 40 = 0$ (d) $14x + 23y - 40 = 0$

47. The equations of transverse and conjugate axes of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$ and their respectively lengths are $\sqrt{2}$ and $2/\sqrt{3}$. The equation of the hyperbola is

(a) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ (b) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

(c) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ (d) $2(2x - y + 4)^2 - (x + 2y - 3)^2 = 1$

48. If $\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to

(a) -4 (b) 1 (c) -7 (d) 5

49. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3, respectively, then the variance of 6 observations $x_1, x_2, ...x_5$ and – 50 is equal to

(a) 507.5 (b) 586.5 (c) 582.5 (d) 509.5

50. Number 1, 2, 3, ..., 100 are written down on each of the cards *A*, *B* and *C*. One number is selected at random from each of the cards. The probability that the numbers so selected can be the measures (in cm) of three sides of right-angled triangles no two of which are similar, is

(a) $\frac{4}{100^3}$ (b) $\frac{3}{50^3}$ (c) $\frac{36}{100^3}$ (d) None of these

OMR SHEET

1	●	b	c	d	**2**	a	●	c	d	**3**	a	b	●	d	**4**	●	b	c	d
5	a	●	c	d	**6**	●	b	c	d	**7**	a	b	●	d	**8**	a	b	c	●
9	a	●	c	d	**10**	a	b	●	d	**11**	a	b	●	d	**12**	a	b	●	d
13	a	●	c	d	**14**	a	●	c	d	**15**	a	b	●	d	**16**	●	b	c	d
17	●	b	c	d	**18**	a	●	c	d	**19**	a	b	●	d	**20**	a	b	●	d
21	●	b	c	d	**22**	a	b	●	d	**23**	a	b	c	●	**24**	●	b	c	d
25	a	●	c	d	**26**	a	●	c	d	**27**	a	●	c	d	**28**	a	●	c	d
29	a	●	c	d	**30**	a	●	c	d	**31**	a	b	●	d	**32**	a	b	c	●
33	a	●	c	d	**34**	a	●	c	d	**35**	●	b	c	d	**36**	a	●	c	d
37	a	b	c	●	**38**	a	b	●	d	**39**	a	●	c	d	**40**	a	●	c	d
41	a	b	●	d	**42**	a	b	c	●	**43**	●	b	c	d	**44**	a	b	●	d
45	●	b	c	d	**46**	a	b	c	●	**47**	a	●	c	d	**48**	a	b	●	d
49	●	b	c	d	**50**	a	b	c	●										

Answers with Hints

1. It is given that $n(A) = m$ and $n(B) = n$

and $$2^m = 2^n + 112$$

[$\because$ number of subsets of set A and B are 2^m and 2^n, respectively]

$$\Rightarrow \quad 2^m - 2^n = 2^4(7)$$

$$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^4(2^3 - 1)$$

On comparing $n = 4$ and $m - n = 3$

$\therefore \quad m = 7$

So, $m \cdot n = 28$

2. $n(A) = 40\%$ of $10000 = 4000$, $n(B) = 2000$,

$n(C) = 1000$, $n(A \cap B) = 500$,

$n(B \cap C) = 300$, $n(C \cap A) = 400$,

$n(A \cap B \cap C) = 200$

$$\therefore \quad n(A \cap \overline{B} \cap \overline{C}) = n\{A \cap (B \cup C)'\} = n(A) - n\{A \cap (B \cup C)\}$$

$$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 4000 - 500 - 400 + 200 = 3300$$

3. $$U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\} = \{0, 1, 2, 3\}$$

$$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$$

and $B = \{x : x^2 - 3x + 2 = 0\} = \{2, 1\}$

$$\therefore \quad (A \cap B)' = U - (A \cap B)$$

$$= \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\}$$

4. Given, $\dfrac{3 - |x|}{4 - |x|} \geq 0 \Rightarrow 3 - |x| \leq 0$ and $4 - |x| < 0$

or $\quad 3 - |x| \geq 0$ and $4 - |x| > 0$

$\Rightarrow \quad |x| \geq 3$ and $|x| > 4$ or $|x| \leq 3$ and $|x| < 4$

$\Rightarrow \quad |x| > 4$ or $|x| \leq 3$

$\Rightarrow \quad (-\infty, -4) \cup [-3, 3] \cup (4, \infty)$

5. $x^{12} - x^9 + x^4 - x + 1 > 0$, three cases arise

Case I When $x \leq 0$, $x^{12} > 0$, $-x^9 > 0$, $x^4 > 0$, $-x > 0$

$$\Rightarrow \quad x^{12} - x^9 + x^4 - x + 1 > 0, \forall\ x \leq 0 \qquad \text{...(i)}$$

Case II When $0 < x \leq 1$,

$$x^9 < x^4, x < 1 \Rightarrow -x^9 + x^4 > 0 \text{ and } 1 - x > 0$$

$$\therefore \quad x^{12} - x^9 + x^4 - x + 1 > 0, \forall\ 0 < x \leq 1 \qquad \text{...(ii)}$$

Case III When $x > 1$, $x^{12} > x^9$, $x^4 > x$

$$\Rightarrow \quad x^{12} - x^9 + x^4 - x + 1 > 0, \forall\ x > 1 \qquad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), the above equation holds for $x \in R$.

6. Given, $\log_{10}(x^3 + y^3) - \log_{10}(x^2 + y^2 - xy) \le 2$

$$\Rightarrow \quad \log_{10} \frac{(x^3 + y^3)}{x^2 + y^2 - xy} \le 2$$

$$\Rightarrow \quad \log_{10}(x + y) \le 2 \Rightarrow x + y \le 100$$

Using AM $\ge$ GM,

$$\frac{x + y}{2} \ge \sqrt{xy} \Rightarrow \sqrt{xy} \le \frac{x + y}{2} \le \frac{100}{2}$$

$$\therefore \quad xy \le 2500$$

7. Given, $3600 = \frac{40}{2}[2a + (40 - 1)d]$

$$\Rightarrow \quad 3600 = 20(2a + 39d)$$

$$\Rightarrow \quad 180 = 2a + 39d \quad \text{...(i)}$$

After 30 instalments one-third of the debt is unpaid.

Hence, $\frac{3600}{3} = 1200$ is unpaid and 2400 is paid.

Now, $2400 = \frac{30}{2}\{2a + (30 - 1)d\}$

$$\therefore \quad 160 = 2a + 29d \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$a = 51, d = 2$$

Now, the value of 8th instalment

$$= a + (8 - 1)d$$

$$= 51 + 7 \cdot 2 = ₹\ 65$$

8. Let $S = 1 + 2 + 3 + \ldots + 100$

$$= \frac{100}{2}(1 + 100) = 50(101) = 5050$$

Let $S_1 = 3 + 6 + 9 + 12 + \ldots + 99$

$$= 3(1 + 2 + 3 + 4 + \ldots + 33)$$

$$= 3 \cdot \frac{33}{2}(1 + 33) = 99 \times 17 = 1683$$

Let $S_2 = 5 + 10 + 15 + \ldots + 100$

$$= 5(1 + 2 + 3 + \ldots + 20)$$

$$= 5 \cdot \frac{20}{2}(1 + 20) = 50 \times 21 = 1050$$

Let $S_3 = 15 + 30 + 45 + \ldots + 90$

$$= 15(1 + 2 + 3 + \ldots + 6)$$

$$= 15 \cdot \frac{6}{2}(1 + 6) = 45 \times 7 = 315$$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$$= 5050 - 1683 - 1050 + 315 = 2632$$

9. Given, $a = 1$ and $4T_2 + 5T_3$ is minimum.

Let r be the common ratio of the GP, then

$\therefore \quad 4T_2 + 5T_3 = 4(ar) + 5(ar^2)$ $\quad [\because a = 1]$

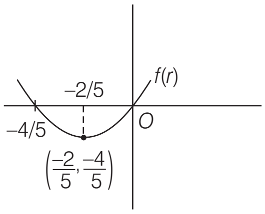
$\Rightarrow \quad 4r + 5r^2 = f(r)$ $\quad$ [say] ...(i)

$\Rightarrow \quad r(4 + 5r) = f(r)$

$f(r) = 0$

$r(4 + 5r) = 0$

$\Rightarrow \quad r = 0, -4/5$



We know that, if $a > 0$, quadratic expression

$$ax^2 + bx + c \text{ has least value at } x = -\frac{b}{2a}.$$

From the graph it is clear that, minima occurs of point $\left(\frac{-2}{5}, \frac{-4}{5}\right)$.

$\therefore \quad r = \frac{-2}{5}$

10. $x^2 + 1 = 0$

$\Rightarrow \quad x = \pm i$

$\because x^2 + 1$ is root of

$P(x) = 5x^4 + 4x^3 + 3x^2 + Mx + N$

Hence, $x = i$ and $-i$ are roots of $P(x)$.

$\Rightarrow \quad P(i) = 0$ and $P(-i) = 0$

$\Rightarrow \quad 5(i)^4 + 4i^3 + 3i^2 + Mi + N = 0$

and $5(-i)^4 + 4(-i)^3 + 3(i)^2 + M(-i) + N = 0$

$\Rightarrow \quad 5 - 4i - 3 + Mi + N = 0$

and $\quad 5 + 4i - 3 - Mi + N = 0$

$\Rightarrow \quad (2 + N) + i(M - 4) = 0$

and $\quad (2 + N) + i(4 - M) = 0$

On comparing real and imaginary parts to zero, we get

$N = -2, M = 4$

and $\quad N = -2, M = 4$

Hence, M and N are unique.

and $M - N = 4 - (-2) = 6$

11. Let $z_1 = r_1 (\cos\theta_1 + i\sin\theta_1)$

and $z_2 = r_2 (\cos\theta_2 + i\sin\theta_2)$

Since, $|z_2| = |z_1|$

$\therefore \qquad r_2 = r_1$

Also, $\arg(z_1) + \arg(z_2) = \pi$

$\therefore \qquad \arg(z_2) = \pi - \arg(z_1)$

$\Rightarrow \qquad \arg(z_2) = \pi - \theta_1$

$\therefore \qquad z_2 = r_1 \{\cos(\pi - \theta_1) + i\sin(\pi - \theta_1)\}$

$= r_1(-\cos\theta_1 + i\sin\theta_1)$

$= -r_1(\cos\theta_1 - i\sin\theta_1) = -\bar{z}_1$

$\Rightarrow \qquad z_1 = -\bar{z}_2$

12. $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} - 3\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$

$\Rightarrow \qquad 4 + 5(\omega)^{334} - 3(-\omega^2)^{365}$

$\Rightarrow \qquad 4 + 5\omega + 3\omega$

$\Rightarrow \qquad \frac{1}{2}\{8 - 5 + 5i\sqrt{3} - 3 + 3i\sqrt{3}\}$

$\Rightarrow \qquad \frac{1}{2}\{8i\sqrt{3}\} = 4\sqrt{3}i$

13. In out of 10 persons, P_1 is always consider and P_4 and P_5 is not consider.

i.e. We have to select, 4 persons out of 7 person and after that they arrange it.

$\therefore$ Required number of ways $= {}^7C_4 \times 5!$

$= \frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 120$

$= 35 \times 120 = 4200$

14. Total number of ways $= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + \ldots + {}^{10}C_{10}$

$= 2^{10} - 1 \qquad [\because {}^nC_0 + {}^nC_1 + {}^nc_2 + {}^nC_n \ldots = 2^n]$

15. Total number of ways = (Attempt 3 from group I and 4 from group II)

+ (Attempt 4 from group I and 3 from group II)

+ (Attempt 5 from group I and 2 from group II)

+ (Attempt 2 from group I and 5 from group II)

$= {}^6C_3 \times {}^6C_4 + {}^6C_4 \times {}^6C_3 + {}^6C_5 \times {}^6C_2 + {}^6C_2 \times {}^6C_5$

$= 2({}^6C_3 \times {}^6C_4) + 2({}^6C_5 \times {}^6C_2)$

$= 2(20 \times 15) + 2(6 \times 15)$

$= 600 + 180 = 780$

16. Given, $A = \{x : x^2 - 5x + 6 = 0\}$

$= \{x : (x-2)(x-3) = 0\} = \{2, 3\}$

and $\quad B = \{2, 4\}$ and $C = \{4, 5\}$

Now, $B \cap C = \{2, 4\} \cap \{4, 5\} = \{4\}$

$\therefore\ A \times (B \cap C) = \{2, 3\} \times \{4\} = \{(2, 4), (3, 4)\}$

17. The general term of $(1 + x)^{2n}$ is $T_{r+1} = {}^{2n}C_r\ x^r$

$$T_2 = {}^{2n}C_1 x^2, T_3 = {}^{2n}C_2 x^3, T_4 = {}^{2n}C_3 x^4$$

Since , coefficients are in AP

$\Rightarrow \quad {}^{2n}C_1, {}^{2n}C_2, {}^{2n}C_3$ are in AP

$$\Rightarrow \quad 2 \times {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$$

$$\Rightarrow \quad 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2}$$

$$\Rightarrow \quad 2 = \frac{2}{(2n - 2 + 1)} + \frac{2n - 3 + 1}{3}$$

$$\Rightarrow \quad 2 = \frac{2}{2n - 1} + \frac{2n - 2}{3}$$

$$\Rightarrow \quad 2n^2 - 9n + 7 = 0$$

$$\therefore \quad 2n^2 - 9n = -7$$

18. $(19)^{2005} + (11)^{2005} - (9)^{2005}$

$$= (10 + 9)^{2005} + (10 + 1)^{2005} - (9)^{2005}$$

$$= \{9^{2005} + {}^{2005}C_1\ (9)^{2004} \times 10 + \ldots\} + ({}^{2005}C_0 + {}^{2005}C_1\ 10 + \ldots) - (9)^{2005}$$

$$= ({}^{2005}C_1\ 9^{2004} \times 10 + \text{multiple of } 10) + (1 + \text{multiple of } 10)$$

$\therefore$ Unit digit = 1

19. Given, $\left(x^2 + \frac{1}{x^2} + 2\right)^n \Rightarrow \left\{\left(x + \frac{1}{x}\right)^2\right\}^n \Rightarrow \left(x + \frac{1}{x}\right)^{2n}$

Here, $2n$ is even.

Therefore, total terms $(2n + 1)$ is odd.

Thus, only one middle term exist.

and $\left(\frac{2n}{2} + 1\right)$ i.e. $(n + 1)$th term will be middle term.

$$\therefore\ T_{n+1} = {}^{2n}C_n\ (x)^{2n-n} \left(\frac{1}{x}\right)^n = {}^{2n}C_n\ x^{2n-2n} = {}^{2n}C_n = \frac{2n!}{n!n!}$$

$$= \frac{(2n)\,(2n-1)\,(2n-2)\,(2n-3)\,(2n-4)\cdots 1}{n!n!}$$

$$= \frac{\{(2n-1)\,(2n-3)\,(2n-5)\cdots 1\} \times \{2n \cdot 2\,(n-1) \cdot 2\,(n-2)\cdots 2\}}{n!n!}$$

$$= \frac{\{(2n-1)\,(2n-3)\,(2n-5)\cdots 1\} \times \{2^n \cdot n\,(n-1)\,(n-2)\cdots 1\}}{n!\,n!}$$

$$= \frac{\{(2n-1)\,(2n-3)\,(2n-5)\cdots 1\}\ 2^n\ n!}{n!\,n!} \qquad [\because n! = n\,(n-1)\,(n-2)\cdots 1]$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\ 2^n}{n!}$$

20. We have, $\cos A = m \cos B$

$\Rightarrow \quad \dfrac{\cos A}{\cos B} = \dfrac{m}{1}$

$\Rightarrow \quad \dfrac{\cos A + \cos B}{\cos A - \cos B} = \dfrac{m+1}{m-1}$

$\Rightarrow \quad \dfrac{2\cos\dfrac{A+B}{2}\cos\dfrac{B-A}{2}}{2\sin\dfrac{A+B}{2}\sin\dfrac{B-A}{2}} = \dfrac{m+1}{m-1}$

$\Rightarrow \quad \cot\dfrac{A+B}{2} = \left(\dfrac{m+1}{m-1}\right)\tan\dfrac{B-A}{2}$

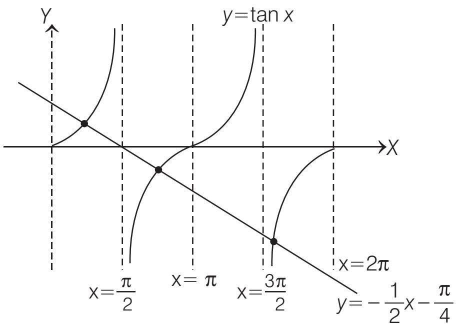
But $\quad \cot\dfrac{A+B}{2} = \lambda \tan\dfrac{B-A}{2}$

$\therefore \quad \lambda = \dfrac{m+1}{m-1}$

21. $x + 2\tan x = \dfrac{\pi}{2}$

$\Rightarrow \quad 2\tan x = \dfrac{\pi}{2} - x$

$\Rightarrow \quad \tan x = -\dfrac{1}{2}x + \dfrac{\pi}{4}$



Number of solutions of the given equation is '3'.

22. We have, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad \cos 60^\circ = \dfrac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad a^2 + b^2 - c^2 = ab$

$\Rightarrow \quad b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Rightarrow \quad b(b+c) + a(a+c) = (a+c)(b+c)$

On dividing by $(a+c)(b+c)$ and add 2 on both sides, we get

$1 + \dfrac{b}{a+c} + 1 + \dfrac{a}{b+c} = 3$

$\Rightarrow \quad \dfrac{1}{a+c} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$

23. Let $O(0, 0)$ be the orthocentre, $A(h, k)$ be the third vertex and $B(-2,3)$ and $C(5,-1)$ the other two vertices. Then, the slope of the line through A and O is $\frac{k}{h}$, while the line through B and C has the slope $\frac{(-1-3)}{(5+2)} = -\frac{4}{7}$. By the poperty of the orthocentre, these two lines must be perpendicular, so we have

$$\left(\frac{k}{h}\right)\left(-\frac{4}{7}\right) = -1$$

$$\Rightarrow \quad \frac{k}{h} = \frac{7}{4} \quad \text{...(i)}$$

Also, $$\frac{5-2+h}{3} + \frac{-1+3+k}{3} = 7$$

$$\Rightarrow \quad h + k = 16 \quad \text{...(ii)}$$

which is not satisfied by the points given in the options (a), (b) or (c).

24. Equation of a line passing through the point of intersection of lines is

$$x - y + 1 + \lambda(2x - 3y + 5) = 0$$

$$\Rightarrow \quad x(1 + 2\lambda) + y(-1 - 3\lambda) + 1 + 5\lambda = 0 \quad \text{...(i)}$$

Its distance from point (3, 2) $= \frac{7}{5}$

$$\Rightarrow \quad \frac{|3(1+2\lambda) + 2(-1-3\lambda) + 1 + 5\lambda|}{\sqrt{(1+2\lambda)^2 + (-1-3\lambda)^2}} = \frac{7}{5} \Rightarrow \frac{|2+5\lambda|}{\sqrt{13\lambda^2 + 10\lambda + 2}} = \frac{7}{5}$$

On squaring, we get

$$25(4 + 25\lambda^2 + 20\lambda) = 49(13\lambda^2 + 10\lambda + 2)$$

$$\Rightarrow \quad 6\lambda^2 - 5\lambda - 1 = 0 \Rightarrow \lambda = 1, -\frac{1}{6}$$

On putting $\lambda = 1, -\frac{1}{6}$ in Eq. (i) respectively, we get

$$3x - 4y + 6 = 0 \text{ and } 4x - 3y + 1 = 0$$

25. The equation of lines are

$$y - y_1 = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}(x - x_1)$$

Since, $m_1 = 1, \quad m_2 = 1$

$$\therefore \quad y - 4 = \frac{1 \pm 1}{1 \mp 1}(x - 3)$$

$$\Rightarrow \quad y = 4 \quad \text{or} \quad x = 3$$

Hence, the lines which make the triangle are $x - y = 2$, $x = 3$ and $y = 4$.

The intersection points of these lines are (6, 4), (3, 1) and (3, 4).

$$\therefore \text{ Area of triangle} = \frac{1}{2}|6(1-4) + 3(4-4) + 3(4-1)|$$

$$= \frac{1}{2}|6(-3) + 3(0) + 3(3)|$$

$$= \frac{1}{2}|-18 + 0 + 9| = \frac{9}{2} \text{ sq units}$$

26. The intersection of line and circle is

$$x^2 + m^2x^2 - 20mx + 90 = 0$$

$\Rightarrow$ $$x^2(1+m^2) - 20mx + 90 = 0$$

Now, $D < 0$ [since, the line does not intersect the circle]

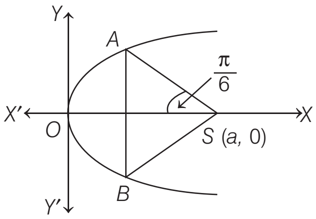
$\Rightarrow$ $$400m^2 - 4 \times 90(1+m^2) < 0$$

$\Rightarrow$ $$40m^2 < 360$$

$\therefore$ $$|m| < 3$$

27. Let $A \equiv (at_1^2, 2at_1), B \equiv (at_2^2, 2at_2)$

We have, $m_{AS} = \tan\left(\frac{5\pi}{6}\right) \Rightarrow \frac{2at_1}{at_1^2 - a} = -\frac{1}{\sqrt{3}}$



$\Rightarrow$ $$t_1^2 + 2\sqrt{3}t_1 - 1 = 0 \Rightarrow t_1 = -\sqrt{3} \pm 2$$

Clearly, $t_1 = -\sqrt{3} - 2$ is rejected.

Thus, $t_1 = (2 - \sqrt{3})$

Hence, $AB = 4at_1 = 4a(2-\sqrt{3})$

28. Normal at $\left(ae, \frac{b^2}{a}\right)$ of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x - ae}{\frac{ae}{a^2}} = \frac{y - \frac{b^2}{a}}{\left(\frac{b^2}{a}/b^2\right)}$$

Since, it passes through $(0, -b)$, then

$$\frac{0 - ae}{\frac{ae}{a^2}} = \frac{-b - \frac{b^2}{a}}{\frac{1}{a}}$$

$\Rightarrow$ $$-a^2 = -a\left(b + \frac{b^2}{a}\right)$$

$\Rightarrow$ $$a^2 = ab + b^2$$

$\Rightarrow$ $$a^2 = ab + a^2 - a^2e^2$$ $[\because b^2 = a^2 - a^2e^2]$

$\Rightarrow$ $$b = ae^2$$

$\Rightarrow$ $$b^2 = a^2e^4$$

$\Rightarrow$ $$a^2(1-e^2) = a^2e^4$$

$\Rightarrow$ $$1 - e^2 = e^4$$

$\Rightarrow$ $$e^2(e^2 + 1) = 1$$

29. Let $P(x, y, z)$ be any point in the plane.

According to the given condition,

$$(\text{distance from } P \text{ to } X\text{-axis})^2 + (\text{distance from } P \text{ to } Y\text{-axis})^2 + (\text{distance from } P \text{ to } Z\text{-axis})^2 = 36$$

$$\Rightarrow \quad (\sqrt{y^2 + z^2})^2 + (\sqrt{x^2 + z^2})^2 + (\sqrt{x^2 + y^2})^2 = 36$$

$$\Rightarrow \quad (y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2) = 36$$

$$\Rightarrow \quad 2(x^2 + y^2 + z^2) = 36$$

$$\Rightarrow \quad x^2 + y^2 + z^2 = 18 \quad \text{...(i)}$$

$\therefore$ The distance from origin to the point (x, y, z) is

$$= \sqrt{x^2 + y^2 + z^2} = \sqrt{18} \quad [\because \text{from Eq. (i)}]$$

$$= 3\sqrt{2}$$

30. Let the fourth vertex be (x, y, z).

We know that, diagonals of a parallelogram are bisecting to each other.

i.e. mid-point of a diagonals are coinciding.

$\therefore$ Mid-point of diagonal AC = Mid-point of diagonal BD

$$\therefore \quad \left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) = \left(\frac{-1+x}{2}, \frac{-2+y}{2}, \frac{-1+z}{2}\right)$$

$$\Rightarrow \quad \left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right) = \left(\frac{-1+x}{2}, \frac{-2+y}{2}, \frac{-1+z}{2}\right)$$

$$\Rightarrow \quad \frac{3}{2} = \frac{-1+x}{2}, \frac{5}{2} = \frac{-2+y}{2}, \frac{5}{2} = \frac{-1+z}{2}$$

$$\Rightarrow \quad x = 4, y = 7, z = 6$$

Hence, required point is (4, 7, 6).

31. Given that,

$$\lim_{x \to \infty} \left[\frac{x^3 + 1}{x^2 + 1} - (ax + b)\right] = 2$$

$$\Rightarrow \lim_{x \to \infty} \left[\frac{x^3(1-a) - bx^2 - ax + (1-b)}{x^2 + 1}\right] = 2$$

$$\Rightarrow \quad \lim_{x \to \infty} \left[\frac{x(1-a) - b - \frac{a}{x} + \frac{(1-b)}{x^2}}{1 + \frac{1}{x^2}}\right] = 2$$

This limit will exist, if

$$1 - a = 0 \quad \text{and} \quad b = -2$$

$$\Rightarrow \quad a = 1 \quad \text{and} \quad b = -2$$

32. $\lim_{x \to \infty} \dfrac{x^4 \cdot \sin\left(\dfrac{1}{x}\right) + x^2}{1 + |x|^3} = \lim_{x \to \infty} \left[\dfrac{x \sin\left(\dfrac{1}{x}\right) + \dfrac{1}{x}}{\dfrac{1}{x^3} + \dfrac{|x|^3}{x^3}} \right]$ [dividing numerator and denominator by x^3]

$$= \frac{\lim_{x \to \infty} \dfrac{\sin\left(\dfrac{1}{x}\right)}{\dfrac{1}{x}} + \lim_{x \to \infty} \dfrac{1}{x}}{\lim_{x \to \infty} \dfrac{1}{x^3} + \lim_{x \to \infty} \dfrac{|x|^3}{x^3}}$$

$$= \frac{1 - 0}{0 + 1} = 1$$

33. $\lim_{n \to \infty} \left(\dfrac{n^2 - n + 1}{n^2 - n - 1} \right)^{n(n-1)}$

$$= \lim_{n \to \infty} \left(\frac{n(n-1) + 1}{n(n-1) - 1} \right)^{n(n-1)}$$

$$= \lim_{n \to \infty} \frac{[n(n-1)]^{n(n-1)} \left[1 + \dfrac{1}{n(n-1)} \right]^{n(n-1)}}{[n(n-1)]^{n(n-1)} \left[1 - \dfrac{1}{n(n-1)} \right]^{n(n-1)}}$$

$$= \lim_{n \to \infty} \frac{\left(1 + \dfrac{1}{n(n-1)} \right)^{n(n-1)}}{\left(1 - \dfrac{1}{n(n-1)} \right)^{n(n-1)}} = \frac{e}{e^{-1}} = e^2 \qquad \left[\because \lim_{n \to \infty} \left(1 + \frac{1}{n} \right)^n = e \right]$$

34. Let H_1 and n_1 are the harmonic mean and number of observations of first group and H_2 and n_2 are the harmonic mean and number of observations of another group.

$\therefore$ By using combined harmonic mean formula, we get

Combined harmonic mean $= \dfrac{n_1 + n_2}{\dfrac{n_1}{H_1} + \dfrac{n_2}{H_2}} = \dfrac{5 + 5}{\dfrac{5}{\frac{5}{2}} + \dfrac{5}{\frac{9}{2}}} = \dfrac{10}{\frac{28}{9}} = \dfrac{90}{28} = \dfrac{45}{14}$

35. Let observations are denoted by x_i for

$$1 \le i \le 2n$$

$$\bar{x} = \frac{\Sigma x_i}{2n} = \frac{(a + a + \ldots + a) - (a + a + \ldots + a)}{2n} = 0$$

and $\sigma_x^2 = \dfrac{\Sigma x_i^2}{2n} - (\bar{x})^2$

$$= \frac{a^2 + a^2 + \ldots + a^2}{2n} - 0$$

$$= a^2$$

$$\sigma_x = a$$

Now, adding a constant b, then

$$\bar{y} = \bar{x} + b = 5$$

$$b = 5$$

and $\sigma_y = \sigma_x$ (No change in SD)

$\Rightarrow$ $a = 20$

$\Rightarrow$ $a^2 + b^2 = (20)^2 + 5^2$

$= 425$

36. The total number of ways $= 6^3 = 216$

If the second number is i $(i > 1)$, then the total number of favourable ways

$$= \sum_{i=1}^{5} (i-1)(6-i) = 20$$

$\therefore$ Required probability $= \dfrac{20}{216} = \dfrac{5}{54}$

37. 13 applicants = 8 men + 5 women

2 persons are selected i.e. (1 men + 1 women) or 2 women

$\therefore$ Required probability $= \dfrac{{}^8C_1 \times {}^5C_1}{{}^{13}C_2} + \dfrac{{}^5C_2}{{}^{13}C_2} = \dfrac{50}{{}^{13}C_2} = \dfrac{25}{39}$

38. Here, $P(A \cup B) = \dfrac{3}{5}$ and $P(A \cap B) = \dfrac{1}{5}$.

So, from the addition theorem,

$$\frac{3}{5} = P(A) + P(B) - \frac{1}{5}$$

or $\dfrac{4}{5} = 1 - P(\bar{A}) + 1 - P(\bar{B})$

$\therefore$ $P(\bar{A}) + P(\bar{B}) = 2 - \dfrac{4}{5} = \dfrac{6}{5}$

39. Let λ be the period of $\sin x + \cos ax$.

Then, $\sin(\lambda + x) + \cos a(\lambda + x) = \sin x + \cos ax \; \forall \; x$ in this identity, putting $x = 0$ and $x = -\lambda$, we get

$$\sin\lambda + \cos a\lambda = 1 \text{ and } 1 = -\sin\lambda + \cos a\lambda$$

On solving above equations, we get

$$\sin\lambda = 0 \text{ and } \cos a\lambda = 1$$

Hence, $\lambda = n\pi$ and $a\lambda = 2m\pi$, where m, n are non-zero integers.

Here, $\dfrac{a\lambda}{\lambda} = \dfrac{2m\pi}{n\pi} \Rightarrow a = \dfrac{2m}{n}$ $[\because \lambda \neq 0]$

40. Given, $f(x) = \tan\sqrt{\dfrac{\pi^2}{9} - x^2}$

For $f(x)$ to be defined,

$$\frac{\pi^2}{9} - x^2 \geq 0$$

$$\Rightarrow \qquad x^2 \le \frac{\pi^2}{9} \quad \Rightarrow \quad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$$

$\therefore$ Domain of $f = \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

Since, the greatest value of $f(x)$ is $\tan\sqrt{\frac{\pi^2}{9} - 0}$, when $x = 0$ and the least value of $f(x)$ is $\tan\sqrt{\frac{\pi^2}{9} - \frac{\pi^2}{9}}$, when $x = \frac{\pi}{3}$.

So, the greatest value of $f(x)$ is $\sqrt{3}$ and the least value of $f(x)$ is 0.

$\therefore$ Range of $f = [0, \sqrt{3}]$

41. Initially the ball falls from a height of 120 m. After striking the floor, it rebounds and goes to a height of $\frac{4}{5} \times (120)$ m. Now, it falls from a height of $\frac{4}{5} \times (120)$ m and after rebounding goes to a height of $\frac{4}{5}\left[\frac{4}{5}(120)\right]$ m. This process is continued till the ball comes to rest.

Hence, the total distance travelled is

$$120 + 2\left[\frac{4}{5}(120) + \left(\frac{4}{5}\right)^2 (120 + \ldots \infty)\right]$$

$$= 120 + 2\left[\frac{\frac{4}{5}(120)}{1 - \frac{4}{5}}\right] = 1080 \text{ m}$$

42. We have, $a = \cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)$

$$\Rightarrow \qquad a^7 = \left[\cos\left(\frac{2\pi}{7}\right) + i\sin\left(\frac{2\pi}{7}\right)\right]^7$$

$$= \cos 2\pi + i\sin 2\pi = 1 \qquad \text{...(i)}$$

Let $\quad S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6) \qquad [\because \alpha = a + a^2 + a^4, \beta = a^3 + a^5 + a^6]$

$$\Rightarrow \qquad S = a + a^2 + a^3 + a^4 + a^5 + a^6 = \frac{a(1 - a^6)}{1 - a}$$

$$\Rightarrow \qquad S = \frac{a - a^7}{1 - a} = \frac{a - 1}{1 - a} = -1 \qquad \text{...(ii)}$$

Let $\quad P = \alpha\beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$$

$$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3 \qquad \text{[from Eq. (i)]}$$

$$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6) = 3 + S$$

$$= 3 - 1 = 2 \qquad \text{[from Eq. (ii)]}$$

Required equation is,

$$x^2 - Sx + P = 0$$

$$\Rightarrow \qquad x^2 + x + 2 = 0$$

43. Since, there are m-men and 2-women and each participant plays two games with every other participant.

$\therefore$ Number of games played by the men between themselves $= 2 \times {}^mC_2$

and the number of games played between the men and the women $= 2 \times {}^mC_1 \times {}^2C_1$

Now, according to the question,

$$2\,{}^mC_2 = 2\,{}^mC_1\;{}^2C_1 + 84$$

$$\Rightarrow \quad \frac{m!}{2!(m-2)!} = m \times 2 + 42$$

$$\Rightarrow \quad m(m-1) = 4m + 84$$

$$\Rightarrow \quad m^2 - m = 4m + 84$$

$$\Rightarrow \quad m^2 - 5m - 84 = 0$$

$$\Rightarrow \quad m^2 - 12m + 7m - 84 = 0$$

$$\Rightarrow \quad m(m-12) + 7(m-12) = 0$$

$$\Rightarrow \quad m = 12 \qquad [\because m > 0]$$

44. We have,

$$\left[2^{\log_2 \sqrt{9^{x-1}+7}} + \frac{1}{2^{(1/5)\log_2(3^{x-1}+1)}}\right]^7$$

$$= \left[\sqrt{9^{x-1}+7} + \frac{1}{(3^{x-1}+1)^{1/5}}\right]^7$$

$$\therefore \quad T_6 = {}^7C_5 \left(\sqrt{9^{x-1}+7}\right)^{7-5}\left[\frac{1}{(3^{x-1}+1)^{1/5}}\right]^5$$

$$= {}^7C_5\,(9^{x-1}+7)\frac{1}{(3^{x-1}+1)}$$

$$\Rightarrow \quad 84 = {}^7C_5\,\frac{(9^{x-1}+7)}{(3^{x-1}+1)}$$

$$\Rightarrow \quad 9^{x-1} + 7 = 4\,(3^{x-1}+1)$$

$$\Rightarrow \quad \frac{3^{2x}}{9} + 7 = 4\left(\frac{3^x}{3} + 1\right)$$

$$\Rightarrow \quad 3^{2x} - 12(3^x) + 27 = 0$$

$$\Rightarrow \quad y^2 - 12y + 27 = 0 \qquad [\text{put } y = 3^x]$$

$$\Rightarrow \quad (y-3)(y-9) = 0$$

$$\Rightarrow \quad y = 3, 9$$

$$\Rightarrow \quad 3^x = 3, 9$$

$$\Rightarrow \quad x = 1, 2$$

45. Since, $\angle QPC = \alpha$

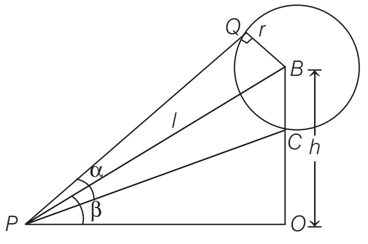
$$\therefore \quad \angle QPB = \angle BPC = \frac{\alpha}{2}$$

In ΔPQB, $\sin\frac{\alpha}{2} = \frac{r}{l}$

$$l = r \text{ cosec } \frac{\alpha}{2} \quad \text{...(i)}$$

and in ΔPOB,

$$\sin\beta = \frac{h}{l}$$



$\Rightarrow$ $h = l\sin\beta$

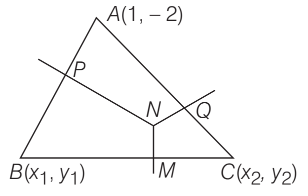
$\Rightarrow$ $h = r\text{cosec}\frac{\alpha}{2}\sin\beta$ [from Eq. (i)]

46. Let $B(x_1, y_1)$ and $C(x_2, y_2)$ are the vertices of a triangle.

$P\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$ lies on the line $x - y + 5 = 0$.

$\therefore$ $\frac{x_1+1}{2} - \frac{y_1-2}{2} = -5$

$\Rightarrow$ $x_1 - y_1 = -13$ ...(i)



Also, $PN \perp AB$

$\therefore$ $\frac{y_1+2}{x_1-1} = -1$

$\Rightarrow$ $y_1 + 2 = -x_1 + 1$

$\Rightarrow$ $x_1 + y_1 = -1$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x_1 = -7$ and $y_1 = 6$

$\therefore$ Coordinates of B are (–7, 6).

Similarly, the coordinates of C are $\left(\frac{11}{5}, \frac{2}{5}\right)$.

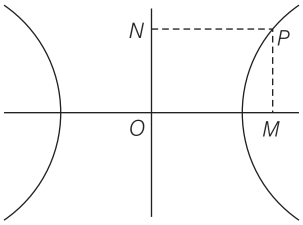
$\therefore$ Equation of BC is $(y - 6) = \frac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$

$\Rightarrow$ $14x + 23y - 40 = 0$

47. We have, transverse axis = $x + 2y - 3 = 0$ and conjugate axis = $2x - y + 4 = 0$ both are perpendicular,

and $\quad 2a = \sqrt{2}$ and $2b = \dfrac{2}{\sqrt{3}}$

$\Rightarrow \quad a = \dfrac{1}{\sqrt{2}}$ and $b = \dfrac{1}{\sqrt{3}}$



We know that,

Equation of the hyperbola referred to two perpendicular lines,

i.e. $$\frac{PN^2}{a^2} - \frac{PM^2}{b^2} = 1$$

$$\Rightarrow \quad \frac{\left(\frac{2x - y + 4}{\sqrt{5}}\right)^2}{\frac{1}{2}} - \frac{\left(\frac{x + 2y - 3}{\sqrt{5}}\right)^2}{\frac{1}{3}} = 1$$

$$\therefore \quad \frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$$

48. It is given that,

$$\lim_{x \to 1} \frac{x^2 - ax + b}{x - 1} = 5 \quad \text{...(i)}$$

Since, limit exist and equal to 5 and denominator is zero at $x = 1$, so numerator $x^2 - ax + b$ should be zero at $x = 1$,

So, $1 - a + b = 0 \Rightarrow a = 1 + b$ ...(ii)

On putting the value of 'a' from Eq. (ii) in Eq. (i), we get

$$\lim_{x \to 1} \frac{x^2 - (1 + b)x + b}{x - 1} = 5$$

$$\Rightarrow \quad \lim_{x \to 1} \frac{(x^2 - x) - b(x - 1)}{x - 1} = 5$$

$$\Rightarrow \quad \lim_{x \to 1} \frac{(x - 1)(x - b)}{x - 1} = 5$$

$$\Rightarrow \quad \lim_{x \to 1} (x - b) = 5$$

$$\Rightarrow \quad 1 - b = 5$$

$$\Rightarrow \quad b = -4 \quad \text{...(iii)}$$

On putting value of 'b' from Eq. (iii) to Eq. (ii), we get

$$a = -3$$

So, $\quad a + b = -7$

49. The mean of five observation,

$$\bar{x} = 10 \qquad \text{(given)}$$

$$\Rightarrow \quad \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 10$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 50 \qquad \text{...(i)}$$

and standard deviation $\text{SD} = \sqrt{\frac{\sum_{i=1}^{5} x_i^2}{5} - (\bar{x})^2} = 3$ (given)

$$\Rightarrow \quad \frac{\sum_{i=1}^{5} x_i^2}{5} - 100 = 9 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5 \times 109$$

$$\Rightarrow \quad \sum_{i=1}^{5} x_i^2 = 545 \qquad \text{... (ii)}$$

Now, variance of 6 observations x_1, x_2, x_3, x_4, x_5 and – 50, is equal to

$$\sigma^2 = \frac{\sum_{i=1}^{5} x_i^2 + (-50)^2}{6} - \left(\frac{\sum_{i=1}^{5} x_i - 50}{6} \right)^2$$

$$= \frac{545 + 2500}{6} - \left(\frac{50 - 50}{6} \right)^2 \qquad \text{[from Eqs. (i) and (ii)]}$$

$$= \frac{3045}{6} = 507.5$$

50. $n(S) = 100 \times 100 \times 100$

We know that,

$$(2n+1)^2 + (2n^2 + 2n)^2 = (2n^2 + 2n + 1)^2 \ \forall \ n \in N.$$

$\therefore$ For $n = 1, 2, 3, 4, 5, 6$, we get lengths of the three sides of a right angled triangle whose longest side ≤ 100.

e.g. When $n = 1$, sides are 3, 4, 5

and when $n = 2$, sides are 5, 12, 13 and so on.

The number of selections of 3, 4, 5 from the three cards by taking one from each is 3!.

$\therefore \quad n(E) = 6\,(3!)$

Hence, $P(E) = \frac{6\,(3!)}{100 \times 100 \times 100} = \frac{1}{100}\left(\frac{3}{50}\right)^2$