Please check the examination details below before entering your candidate information

Candidate Name	Class	Section

BLOOM Mathematics Olympiad (BMO)

Question Paper 2024-25

Class 12

Total Questions: **50 + 5** (Tie-Breaking Section)

Total Time Allotted :	Total Marks
60 minutes	60

Instructions

1. There are **50 Multiple Choice Questions** in this booklet having 4 options out of which **ONLY ONE** is correct.
2. There are two sections in the Question Paper; **Section A** having 40 Questions carrying 1 Mark each & **Section B** having 10 Higher Difficulty Order Questions carrying 2 Marks each.
3. All questions are compulsory. There is **NO negative** marking for incorrect answers.
4. Total time allotted to complete the paper is 60 minutes.
5. Please fill in your details in the space provided on this page before attempting the paper.

OMR Sheet Instructions

1. Before starting the paper, fill in all the details in the OMR sheet.
2. Additional 10 minutes will be provided to fill up the OMR sheet, before the start of the exam.
3. Use HB Pencil to darken the circle of the correct Option in OMR sheet. The correct way to darken the circle in OMR sheet is shown below

   ● ○ ○ ○

4. Use black or blue ball point pen/HB pencil to fill the information in the OMR sheet. Partially filled OMR sheet will not be checked.
5. Return the OMR sheet to the invigilator after the exam.

CODE #121

M12

BLOOM CAP
Founded by arihant

Section A (1 Mark)

1. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then, the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

(a) 717 (b) 720
(c) 763 (d) 840

2. Let $A = R - \{3\}$ and $B = R - \{1\}$. If $f : A \to B$, then, $f(x) = \dfrac{(x-2)}{(x-3)}$ is

(a) one-one and into (b) one-one and onto
(c) many-one and into (d) many-one and onto

3. If $f : R \to R$ and $g : R \to R$ are two functions such that $f(x) = 2x - 3$, $g(x) = x^3 + 5$, then function $(fog)^{-1}(x)$ is equal to

(a) $\left(\dfrac{x+7}{2}\right)^{1/3}$ (b) $\left(x - \dfrac{7}{2}\right)^{1/3}$

(c) $\left(\dfrac{x-2}{7}\right)^{1/3}$ (d) $\left(\dfrac{x-7}{2}\right)^{1/3}$

4. Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi$, $x \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$. Then, $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$ is equal to

(a) $\pi - 2\sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$

(b) $\dfrac{-2\pi}{3}$

(c) 0

(d) $\pi - \sin^{-1}\left(\dfrac{\sqrt{3}}{4}\right)$

5. If the domain of the function $f(x) = \sin^{-1}\left(\dfrac{x-1}{2x+3}\right)$ is $R - (\alpha, \beta)$, then $12\alpha\beta$ is equal to

(a) 36 (b) 32 (c) 40 (d) 24

6. For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha+\beta+\gamma)(\alpha-\gamma+\beta) = 3\alpha\beta$, then γ equals

(a) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ (b) $\sqrt{3}$ (c) $\dfrac{\sqrt{3}}{2}$ (d) $\dfrac{1}{\sqrt{2}}$

7. The greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively.

(a) $\dfrac{5\pi^2}{4}$ and $\dfrac{\pi^2}{8}$ (b) $\dfrac{\pi}{2}$ and $\dfrac{-\pi}{2}$

(c) $\dfrac{\pi^2}{4}$ and $\dfrac{-\pi^2}{4}$ (d) $\dfrac{\pi^2}{4}$ and 0

8. The sum of possible values of x for $\tan^{-1}(x+1) + \cot^{-1}\left(\dfrac{1}{x-1}\right) = \tan^{-1}\left(\dfrac{8}{31}\right)$ is

(a) $\dfrac{-32}{4}$ (b) $-\dfrac{31}{4}$

(c) $-\dfrac{30}{4}$ (d) $-\dfrac{33}{4}$

9. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number and $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is

(a) 2 (b) 3
(c) 1 (d) 4

10. Let $A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$, where $i = \sqrt{-1}$.

Then, the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n = A\}$ is

(a) 10 (b) 20
(c) 25 (d) 30

11. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$.

If $a_{11} = 109$, then a_{22} is equal to

(a) 6 (b) 10
(c) 15 (d) 12

12. Let $A=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}$ and $B=7A^{20}-20A^{7}+2I$, where I is an identity matrix of order 3×3. If $B=[b_{ij}]$, then b_{13} is equal to ....

(a) 900 (b) 800 (c) 910 (d) 810

13. If the matrix $A=\begin{bmatrix}0 & 2\\ K & -1\end{bmatrix}$ satisfies $A(A^3+3I)=2I$, then the value of K is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) -1 (d) 1

14. Let $A=\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$. If $A^{-1}=\alpha I+\beta A,\ \alpha,\beta\in R$, where I is a 2×2 identity matrix, then $4(\alpha-\beta)$ is equal to

(a) 8/3 (b) 4 (c) 2 (d) 5

15. Let A be a 3×3 matrix such that $A^2-5A+7I=0$

Statement I $A^{-1}=\frac{1}{7}(5I-A)$

Statement II The polynomial A^3-2A^2-3A+I can be reduced to $5(A-4I)$. Then,

(a) Both the statements are true.
(b) Both the statements are false.
(c) Statement I is true but Statement II is false.
(d) Statement I is false but Statement II is true.

16. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2=3A+\alpha I$. If $A^4=21A+\beta I$, then

(a) $\alpha=1$ (b) $\alpha=4$
(c) $\beta=8$ (d) $\beta=-8$

17. If $A=\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of α is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

18. Let $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj}\,2A))|=(16)^n$, then n is equal to

(a) 8 (b) 10
(c) 12 (d) 9

19. Let $A(a,0)$, $B(b,2b+1)$ and $C(0,b)$, $b\neq 0, |b|\neq 1$, be points such that the area of ΔABC is 1 sq unit, then the sum of all possible values of a is

(a) $\frac{-2b}{b+1}$ (b) $\frac{2b}{b+1}$
(c) $\frac{2b^2}{b+1}$ (d) $\frac{-2b^2}{b+1}$

20. For what value of k,
$f(x)=\begin{cases}\dfrac{x^3+x^2-16x+20}{(x-2)^2}, & x\neq 2\\ k, & x=2\end{cases}$ is continuous at $x=2$?

(a) 6 (b) 2
(c) 5 (d) 7

21. Let $f:(-\infty,\infty)-\{0\}\to R$ be a differentiable function such that $f'(1)=\lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$. Then,
$\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\log_e a$ is equal to

(a) $\frac{5}{2}+\frac{\pi}{8}$ (b) $\frac{3}{2}+\frac{\pi}{4}$
(c) $\frac{3}{4}+\frac{\pi}{8}$ (d) $\frac{3}{8}+\frac{\pi}{4}$

22. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape, then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is

(a) 18 (b) 10
(c) 9 (d) 20

23. The number of points, where the curve $y=x^5-20x^3+50x+2$ crosses the X-axis, is

(a) 5 (b) 4 (c) 3 (d) 2

24. The function

$$f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$$

(a) increases in $\left[\frac{1}{2}, \infty\right)$

(b) decreases in $\left[\frac{1}{2}, \infty\right)$

(c) increases in $\left(-\infty, \frac{1}{2}\right]$

(d) decreases in $\left(-\infty, \frac{1}{2}\right]$

25. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in

(a) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(b) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(c) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(d) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

26. If $\int \frac{dx}{2\sin^2 x + 5\cos^2 x} = \frac{1}{\sqrt{10}} \tan^{-1}\left(\frac{a \tan x}{b}\right) + C$, then the value of $(a \cdot b)^2$ is

(a) $\sqrt{5}$ (b) $\sqrt{2}$

(c) $\sqrt{10}$ (d) 10

27. If $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{a} \log(b + c)$, then the value of $(ab + c)$ is

(a) 2 (b) 3 (c) 4 (d) 10

28. If $\int \frac{dx}{\cos^3 x \sqrt{2\sin 2x}} = (\tan x)^A + C(\tan x)^B + k$, where k is a constant of integration, then $A + B + C$ equals

(a) $\frac{16}{5}$ (b) $\frac{27}{10}$

(c) $\frac{7}{10}$ (d) $\frac{21}{5}$

29. $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + C$, where c is a constant of integration, then the ordered pair (a, b) is equal to

(a) (3, 1) (b) (1, 3)

(c) (– 1, 3) (d) (1, – 3)

30. Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$, $m, n > 0$. If $\int_0^1 (1 - x^{10})^{20} dx = a \times \beta(b, c)$, then $100(a + b + c)$ equals

(a) 2120 (b) 2012

(c) 1120 (d) 1021

31. If $\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, where, a, b and c are rational numbers, then $2a + 3b - 4c$ is equal to

(a) 8 (b) 10 (c) 7 (d) 4

32. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations $AX = B$ has a negative solution (i.e. $x < 0$ and $y < 0$), be the interval (a, b). Then, $8\int_a^b |A|\, dm$ is equal to

(a) 150 (b) 450

(c) 100 (d) 130

33. For what value of k, the area of the region bounded by the parabola $y^2 = 2x$ and the straight line $x - y = 4$ is $6k$?

(a) $k = 1$ (b) $k = 2$

(c) $k = 3$ (d) $k = 4$

34. What is the area of the region bounded by the curves $x = at^2$ and $y = 2at$ between the ordinates corresponding to $t = 1$ and $t = 2$?

(a) $\frac{28}{3} a^2$ sq units (b) $\frac{56}{3} a^2$ sq units

(c) $\frac{22}{3} a$ sq units (d) $\frac{56}{3} a$ sq units

35. The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to

(a) $\frac{16}{3}$ Sq units (b) $\frac{2}{3}$ Sq units
(c) $\frac{9}{2}$ Sq units (d) $\frac{4}{3}$ Sq units

36. The temperature $T(t)$ of a body at time $t = 0$ is 160°F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$, where K is a positive constant. If $T(15) = 120$°F, then $T(45)$ is equal to

(a) 85°F
(b) 95°F
(c) 80°F
(d) 90°F

37. Let α be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to

(a) 5 (b) 9
(c) 7 (d) 3

38. Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then, $y(2)$ is equal to

(a) 15 (b) 11
(c) 13 (d) 17

39. Let $f(x) = \int_0^x e^t f(t)\,dt + e^x$ be a differentiable function for all $x \in R$. Then, $f(x)$ equals

(a) $2e^{(e^x - 1)} - 1$
(b) $e^{e^x} - 1$
(c) $2e^{e^x} - 1$
(d) $e^{(e^x - 1)}$

40. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x tf(t)dt$, $f(1) = \frac{2}{3}$. Then, $18f(3)$ is equal to

(a) 210 (b) 160 (c) 180 (d) 130

Section B (2 Marks)

41. For any vector $\vec{a} = \vec{a}_1\hat{i} + \vec{a}_2\hat{j} + \vec{a}_3\hat{k}$, with $10|\vec{a}_i| < 1$, $i = 1, 2, 3$, consider the following statements

(A) $\max\{|\vec{a}_1|, |\vec{a}_2|, |\vec{a}_3|\} \le |\vec{a}|$

(B) $|\vec{a}| \le 3\max\{|\vec{a}_1|, |\vec{a}_2|, |\vec{a}_3|\}$

(a) Neither (A) nor (B) is true
(b) Both (A) and (B) are true
(c) Only (B) is true
(d) Only (A) is true

42. Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is

(a) 4 (b) –3 (c) 3 (d) –4

43. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b}$ and $\vec{c}$ be two non-zero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statements

(A) $|\vec{a} + \lambda\vec{c}| \ge |\vec{a}|, \forall \lambda \in R$

(B) $\vec{a}$ and $\vec{c}$ are always parallel.

Then,

(a) Neither (A) nor (B) is correct.
(b) Both (A) and (B) are correct.
(c) only (B) is correct.
(d) only (A) is correct.

44. Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to

(a) 3 (b) 4 (c) 5 (d) 6

45. If two straight line whose direction cosines are given by the relations $l + m - n = 0$, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is

(a) 6 (b) 4
(c) 3 (d) 2

46. If the shortest distance between the lines

$L_1 : \vec{r} = (2+\lambda)\hat{i} + (1-3\lambda)\hat{j} + (3+4\lambda)\hat{k},\ \lambda \in R$

$L_2 : \vec{r} = 2(1+\mu)\hat{i} + 3(1+\mu)\hat{j} + (5+\mu)\hat{k},\ \mu \in R$ is $\frac{m}{\sqrt{n}}$, where $\gcd(m, n) = 1$, then the value of $m + n$ equals

(a) 387 (b) 377
(c) 384 (d) 390

47. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(1, 6, 4)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Then, $2\alpha + \beta + \gamma$ is equal to

(a) 11 (b) 12 (c) 14 (d) 10

48. The probability that a relation R from $\{x,y\}$ to $\{x,y\}$ is both symmetric and transitive, is equal to

(a) $\frac{5}{16}$ (b) $\frac{9}{16}$
(c) $\frac{11}{16}$ (d) $\frac{13}{16}$

49. Let A and B be two non-null events such that $A \subset B$. Then, which of the following statements is always correct?

(a) $P(A/B) = P(B) - P(A)$
(b) $P(A/B) \geq P(A)$
(c) $P(A/B) \leq P(A)$
(d) $P(A/B) = 1$

50. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is m/n, where m and n are coprime, then $4m - 3n$ is equal to

(a) 10 (b) 12 (c) 6 (d) 8

Tie-Breaking Section

Instructions

1. This section consists of 5 questions.
2. The score achieved in this section will not be included in the total marks.
3. If overall marks of two or more students are same, winner will be decided based on the score in this section.
4. Participation in this section is optional and students may choose to attempt it or not.

1. Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = x^2 + xf'(2) + f''(3)$, then which option is correct?

(a) $f'(1) = 4 - f'(2)$ (b) $g'(2) = 8 - g'(1)$
(c) $g''(2) + f''(3) = 4$ (d) None of these

2. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$, $(x \geq 0)$, $f(0) = 0$ and $f(1) = \frac{1}{k}$, then the value of k is

(a) 5 (b) 3
(c) 2 (d) 4

3. Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)dy = y(x - y)\,dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then

(a) $e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$ (b) $e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2})$
(c) $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$ (d) $e^{3\beta^{-1}} = e(5 + \sqrt{2})$

4. If the shortest distance between the straight lines $3(x - 1) = 6(y - 2) = 2(z - 1)$ and $4(x - 2) = 2(y - \lambda) = (z - 3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to

(a) 3 (b) 2 (c) 5 (d) –1

5. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line $\frac{x - 0}{1} = \frac{y - 3}{1} = \frac{z - 1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the ΔPQR is λ and $\lambda^2 = 14K$, then K equal to

(a) 18 (b) 72 (c) 36 (d) 81